Why is it so hard?

I think that anyone who’s ever had a go at a Rubiks cube has had the thought “ If I turn it enough I will eventually solve it” unfortunately for them this is really not something the average human has enough time for.

To find out quite how unlikely that would be we have to look at the mathematical branches of combinatorics and probability.

Combinatorics is responsible for finding how many different combinations there can be on the cube , and probability for finding the likelihood of these combinations happening.

The first step in this problem is figuring out in total how many ways a Rubiks cube can be turned, first of all a small problem to understand the concepts. If we have 6 people and 6 chairs how many different ways can the people be arranged sitting down?

The first person has 6 possible places to choose from, the second only has 5 due to one being taken by the first person, and the third only has 4 and so on. So the number of different ways is or 6!. Which isn’t a very enthusiastic 6 but is instead 6 factorial. In mathematical times the factorial, . And so 6 people can be arranged on 6 chairs in 720 different ways.

This same principle can be applied to the pieces of the cube, there is 3 visible types of piece, a corner with 3 stickers an edge which has 2 and the centre pieces which each have only 1. Luckily for us we can simplify the problem because we don’t have to worry about the centre pieces as they cant move relative to each other. Which means we just have the 8 corners and the 12 edges. And so these can each be arranged in 12! And 8! Ways.

This is known as permutations. The other principle is orientation , with permutations being about how the pieces can be moved orientation is about how they can be rotated. Each corner has 3 stickers, which means it can be oriented 3 different ways, so for all 8 corners each rotating 3 ways the number of orientations for the corners is . And the same for the edges, each can be rotated 2 ways and there’s 12 of them so different orientations.

Sticking all of this together gives us different combinations, that’s the same as (I assume most people reading this will be familiar with standard form , if not you can imagine it as moving the decimal point 20 places back or roughly 5 with 20 0’s after it, 500000000000000000000) It’s a big number, roughly 5 times the number of grains of sand on the earth. However luckily for us this number is incorrect, its slightly too high. In fact it is exactly 12 times too large. This number is the total number of positions possible if you were to take your cube apart and reassemble it however not all of these positions are possible just by turning the layers. For example it isn’t possible to rotate just one corner piece on the cube so if you were to take apart the cube and put a corner piece in rotated the wrong way you now have a whole new set of positions not before possible, this is known as an orbit. The cube has 12 of these orbits, 2 due to it being impossible for one edge to be flipped, 3 for it being impossible for one corner to be rotated and finally there is no way to swap just two edge pieces, so if you were to take out two edge pieces there is 2 ways you can put them back in. This leaves us with different orbits, so there is 12 different sets of possible combinations, and so we must divide our original answer by 12.

Leaving us with the number of possible combinations on a Rubiks cube being

That’s a big number, a really big number. I said earlier that it would be unlikely for anyone to be able to randomly turn the cube and solve it, and so now we can find out quite how unlikely. There’s a general formula for the probability of an event occurring and unfortunately there is only one way to solve it , which makes the probability for every turns you make 1 of them will be correct .

I would like to take some time to give some examples of how big 43,252,003,274,489,856,000 really is.

* Even if you could make 1 turn every nano-second (0.000000001 seconds) it would still take 1371 years to go through every position, or if you prefer to keep it in the family, that’s 49 generations.
* Using a standard sized Rubiks cube you could cover the earth 273 times with every position, only one of those would be solved.
* If you drew every position on a separate piece of paper, and stacked that paper you would be able to stack it to the sun, and then when on the sun ( we’re using heatproof paper) turn around and create a new stack to the earth. You would actually be able to make 14456 round trips. Out of all those stacks one piece of paper has a solved Rubiks cube on it.

It’s a really really big number.

And that what makes this such a recognizably hard puzzle, the mission is to find the one solution out of this huge number.

Mathematicians like to be able to simplify a problem , reduce it to its most basic form. In most cases this results in an equation. We have managed to find out the number of possible combinations for a standard 3x3x3 rubiks cube (3x3x3 describes the number of pieces on each side) but what if we wanted to do the same thing for a 4x4x4, 7x7x7 or even the world record 17x17x17 how could we go about it. Unfortunately we cant just use the same method as before as each cube acts differently, e.g whilst odd cubes have centers which don’t move, any even ones do and this means we have to take their movement into account.

Luckily for us Chris Hardwick former 5x5 and 6x6 blindfold world record holder has already solved the problem, he has come up with a formula to find the possible number of combinations of any nxnxn Rubiks cube.

Most of the functions in this equation should be familiar to you in this equation now, such as the factorial (!) and the exponentials (e.g., however I do expect there to be two which aren’t common knowledge.

- the modulo function is a way of finding the remainder of a number when divided by another number, in our case 2 being the number n is divided by, e.g if n = 7, has a remainder of 1 ,so 7 mod 2 = 1. 10 mod 4 would be 2 because 8 is the closest multiple of 4 and it has a distance of 2 from 10. Going back to our example of n mod 2 we can either get an answer of 1 or 0, if n is odd n mod 2 = 1 , if n is even then it is a multiple of 2 so n mod 2=0.

**–** these brackets missing the top ‘bars’ represents the floor function, also known as the greatest integer function, = the greatest integer lower than x. e.g. , etc, this ones pretty damn simple, take the number at the front.

Now we have this formula lets use it to check our answer,

So for a 3x3x3 cube n=3

Which simplifies to

= 43252003274489856000 which shows we were right.

We now know how difficult the rubiks cube is really, a natural progression would be to look at some bigger cubes now, how about a 7x7x7

Well, this ones a little bit bigger, the number of combinations on a 7x7x7 rubiks cube is

19500551183731307835329126754019748794904992692043434567152132912323232706135469180065278712755853360682328551719137311299993600000000000000000000000000000000000

Or if you want to be able to say that it’s 19 duoquinquagintillion..

And just for the hell of it, the world record 17x17x17 Rubiks cube created by Dutch puzzle maker Oskar Van Deventer.

This one needs a bit more room; 66909260871052009626140831457599196711140812269154070729060136529449625780211961895693820570513604163602868942801633627363413148772664738570971988412147490850469267091069898537146037768890069934919884249763818629080668367898685033459370133844075322446474048403397592421266564641031053781182835951043902666703934718275733629773072428119603386280810232743294106725017906015726602505404809355600713515400760343408510054774806467063695824637124911945446317465833055520836975861238244940397333234336971270687092383804133631886114309853819332336282986834777948178464656888802372250927074981140246608824577036094710201099095240641256513217598802423874027822421584587650039125516202912205481540427864199947576722221866866102507350876922115628881880203115212216766503665426445956786264399133302962649600884736000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000

That’s is a 1055 digit long number, good luck solving that by luck.